

QP.No. : BB 774

4SCJ

B.Sc. B.Ed. (4 — YEAR INTEGRATED)
DEGREE EXAMINATION, JULY 2020.

(For the candidates admitted during the Academic Year
2017–2018 onwards)

Fourth Semester

Core Paper — VIII — REAL ANALYSIS — II

Time : 3 hours

Maximum Marks : 70

PART A — ($10 \times 1 = 10$ marks)

Answer ALL the questions.

1. Define pointwise convergence of sequence of functions.
2. State Dini's Theorem.
3. Let $T : M \rightarrow M$ be a contraction mapping. Prove that T is continuous on M .
4. Say True or False : Any Contraction mapping T defined on a complete metric space has a unique fixed point.
5. Define completion of a metric space.
6. Define a Cauchy Sequence in a metric space.
7. State Chain Rule.
8. State Rolle's Theorem.
9. Define Riemann integrable of function.
10. State the second fundamental theorem of Calculus.

PART B — ($5 \times 4 = 20$ marks)

Answer ALL the questions.

11. Prove that $f_n(x) = \frac{x}{1+nx}$ converges to 0 in $[0, \infty)$.
12. If $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T(x) = \frac{x}{3} \quad \forall \quad x = (x_1, x_2) \in \mathbb{R}^2$,
then prove that T is a contraction on $\mathbb{R}^2$.
13. Prove that any discrete metric space is complete.
14. Find a suitable point C of Rolle's theorem for
 $f(x) = (x-a)(x-b)$, $(a \leq x \leq b)$.
15. Find the Taylor series about $x = 2$ for $f(x) = x^3 + 2x + 1$.

PART C — ($5 \times 8 = 40$ marks)

Answer any FIVE questions.

16. Prove that the series $\sum_{n=1}^{\infty} \frac{nx^2}{n^2 + x^2}$ is uniformly convergent
on $[0, k]$ where $k > 0$.
17. State and prove Cauchy criterion for uniform
convergence.
18. State and prove the contraction mapping theorem.
19. Prove that $\mathbb{R}^n$ with usual metric is complete.
20. State and prove mean value theorem.

21. Prove that the function $f(x)=|x|$ is continuous at $x=0$ but not differentiable at $x=0$.
22. Verify second mean value theorem for the function $f(x)=\sin x$ and $g(x)=e^x$, $x \in (0, \pi]$.
23. If $f \in R[a, b]$ and $a < c < b$ then $f \in R[a, c]$, $f \in R[c, b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$.
-