

**QP.No. : BB 773**

**4SCI**

B.Sc. B.Ed. (4 — YEAR INTEGRATED)  
DEGREE EXAMINATION, JULY 2020.

(For the candidates admitted during the Academic Year  
2017–2018 onwards)

Fourth Semester

Core Paper : VII — LINEAR ALGEBRA AND BOOLEAN  
ALGEBRA

Time : 3 hours

Maximum Marks : 70

PART A — ( $10 \times 1 = 10$  marks)

Answer ALL questions.

1. Define subspace.
2. Define homomorphism.
3. Define norm of an element  $x$ , where  $x \in V$ .
4. Let  $V$  be an inner product space and  $x, y \in V$ . When will you say that  $x$  is orthogonal to  $y$ ?
5. Check whether  $f(x, y) = 1$  is a bilinear form on  $V_2(R)$ , where  $x = (x_1, x_2)$  and  $y = (y_1, y_2)$ .
6. Define symmetric bilinear form.
7. Find the greatest element for the set of all non-empty subsets of a non-empty set  $A$ .
8. Define Lattice.

9. State the commutative law in Boolean Algebra.
10. Define Switch.

PART B — ( $5 \times 4 = 20$  marks)

Answer ALL questions.

11. Prove that  $W = \{(a, 0, 0) / a \in R\}$  is a subspace of  $R^3$ .
12. If  $S$  is any subset of  $V$ , then prove that  $S^\perp$  is a subspace of  $V$ .
13. Find the matrix of  $f$ , if  $f(x, y) = x_1y_1 + x_2y_2$  where  $x = (x_1, x_2)$  and  $y = (y_1, y_2)$  with respect to the standard basis  $\{e_1, e_2\}$ .
14. Let  $L$  be a Lattice. Let  $a, b, c, d \in L$ , then prove that  $a \leq b$  and  $c \leq d \Rightarrow a \vee c \leq b \vee d$ .
15. Construct truth table for the Boolean expression  $x(y + x')$ .

PART C — ( $5 \times 8 = 40$  marks)

Answer any FIVE questions.

16. Prove that the intersection of two subspace of a vector space is a subspace.
17. Let  $V$  be a vector space over a field  $F$ . Let  $A$  and  $B$  be subspace of  $V$ , then prove that  $\frac{A+B}{A} \cong \frac{B}{A \cap B}$ .
18. Let  $S = \{v_1, v_2, \dots, v_n\}$  be an orthogonal set of non-zero vectors in  $V$ . Let  $v \in V$  and  $v = \{\alpha_1v_1 + \alpha_2v_2 + \dots + \alpha_nv_n\}$ .  
Prove that  $\alpha_k = \frac{\langle v, v_k \rangle}{\|v_k\|^2}$ .

19. Let  $f$  be a symmetric bilinear form defined on  $V$ . Let  $q$  be the associated quadratic form. Prove that  $f(u, v) = \frac{1}{4}[q(u+v) - q(u-v)]$ .
20. Reduce the quadratic form  $x_1^2 + 4x_1x_2 + 4x_1x_3 + 4x_2^2 + 16x_2x_3 + 4x_3^2$  to the diagonal form.
21. Show that in any Lattice  $L_5 : a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$  and  $L_5' : a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$  are equivalent.
22. Express the Boolean Expression  $x' + yz$  in CNF in the variables  $x, y, z$ .
23. A committee consisting of 3 members approves any proposal by majority vote. Each member can approve a proposal by pressing a button attached to their seats. Design as simple a circuit as you can which will allow current to pass when and only when a proposal is approved.

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